ME 321: FLUID MECHANICS-I

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Lecture-10

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Measurements in Fluid mechanics

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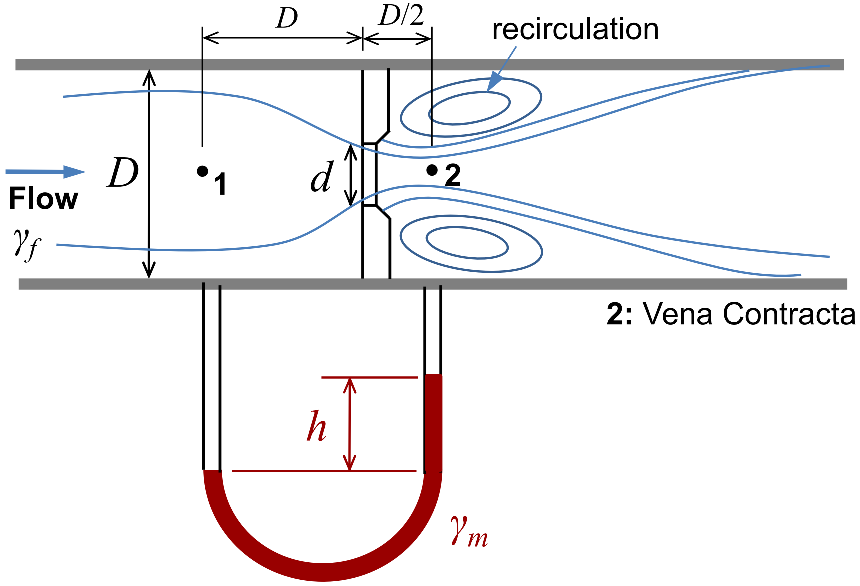
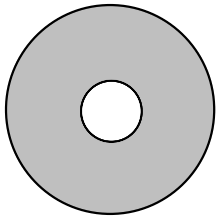
Measurement of

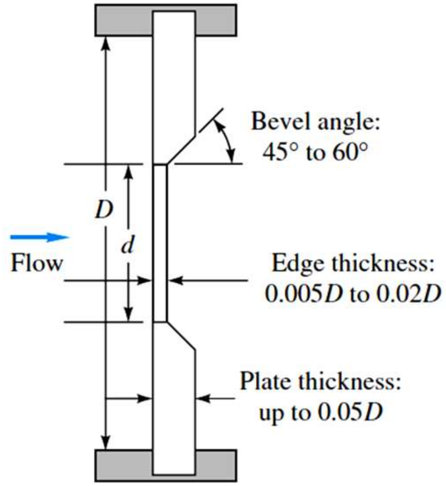
- flow rate (volume flow rate)
- flow velocity

Orifice meter





Orifice plate



ISO standard Orifice plate

Continuity equation: $Q = \frac{\pi}{4} D^2 V_1 = \frac{\pi}{4} d_2{}^2 V_2$

Bernoulli equation between point 1 and 2:

$$\frac{p_1}{\gamma_f} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma_f} + \frac{V_2^2}{2g} + z_2$$

$$\Rightarrow \frac{p_1}{\gamma_f} + \frac{V_1^2}{2g} = \frac{p_2}{\gamma_f} + \frac{V_2^2}{2g}$$

Orifice meter

$$\Rightarrow \frac{V_2^2 - V_1^2}{2g} = \frac{p_1 - p_2}{\gamma_f}$$

$$\Rightarrow V_2^2 - V_1^2 = \frac{2(p_1 - p_2)}{\rho_f}$$

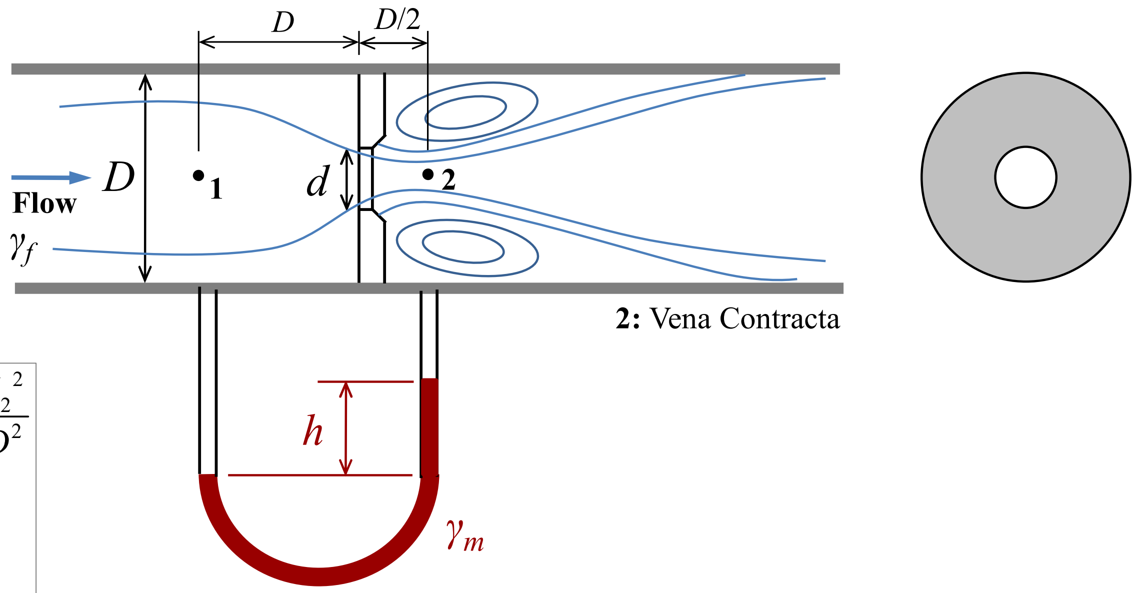
$$\Rightarrow V_2^2\left(1 - \frac{V_1^2}{V_2^2}\right) = \frac{2(p_1 - p_2)}{\rho_f}$$

$$\Rightarrow V_2^2\left(1 - \frac{d_2^{\,4}}{D^4}\right) = \frac{2(p_1 - p_2)}{\rho_f}$$

$$\Rightarrow V_2 = \sqrt{\frac{2(p_1 - p_2)}{\rho_f\left(1 - \frac{d_2^{\,4}}{D^4}\right)}}$$

$$Q = \frac{\pi}{4}D^2 V_1 = \frac{\pi}{4}d_2^{\,2} V_2 \quad \therefore \frac{V_1}{V_2} = \frac{d_2^{\,2}}{D^2}$$

$$\therefore V_1 = \frac{Q}{\frac{\pi}{4}D^2} \quad \text{and} \quad V_2 = \frac{Q}{\frac{\pi}{4}d_2^{\,2}}$$



Then the theoretical volume flowrate could be expressed as:

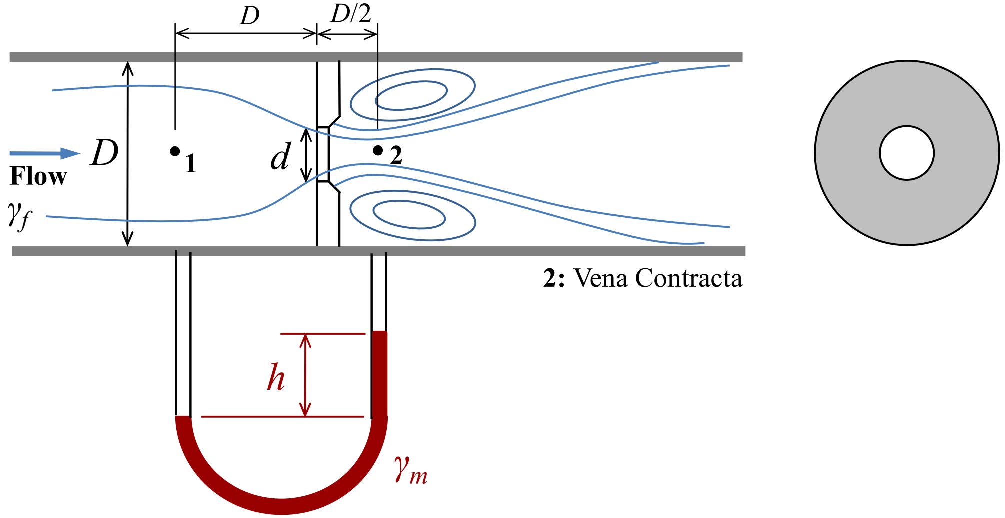
$$Q_{theo.} = A_2 V_2 = \frac{A_2}{\sqrt{\left(1 - \frac{d_2^{\,4}}{D^4}\right)}}\sqrt{\frac{2(p_1 - p_2)}{\rho_f}} \qquad (i)$$

Orifice meter

It is not convenient to use point 2 (vena contracta), instead orifice diameter, *d* is used which could be available from geometric configuration.

Define a coefficient, namely "coefficient of contraction, C_c" as:

$$C_c = \frac{A_2}{A_0} = \frac{d_2^2}{d^2} \left(\equiv \frac{\text{area at vena contracta}}{\text{area at the orifice}} \right)$$



Then Eq. (i) comes as:

$$Q_{theo.} = \frac{C_c A_0}{\sqrt{\left(1 - \frac{C_c^{\,2} d^4}{D^4}\right)}} \sqrt{\frac{2(p_1 - p_2)}{\rho_f}} \qquad (ii)$$

However, assuming $C_c \approx 1$, then Eq. (ii) can be written as:

$$Q_{theo.} \approx \frac{A_0}{\sqrt{\left(1 - \frac{d^4}{D^4}\right)}} \sqrt{\frac{2(p_1 - p_2)}{\rho_f}} \qquad (iii)$$

$$Q_{theo.} \approx \frac{A_0}{\sqrt{\left(1 - \beta^4\right)}} \sqrt{\frac{2(p_1 - p_2)}{\rho_f}} \qquad (iv)$$

where $\beta = \dfrac{d}{D}$

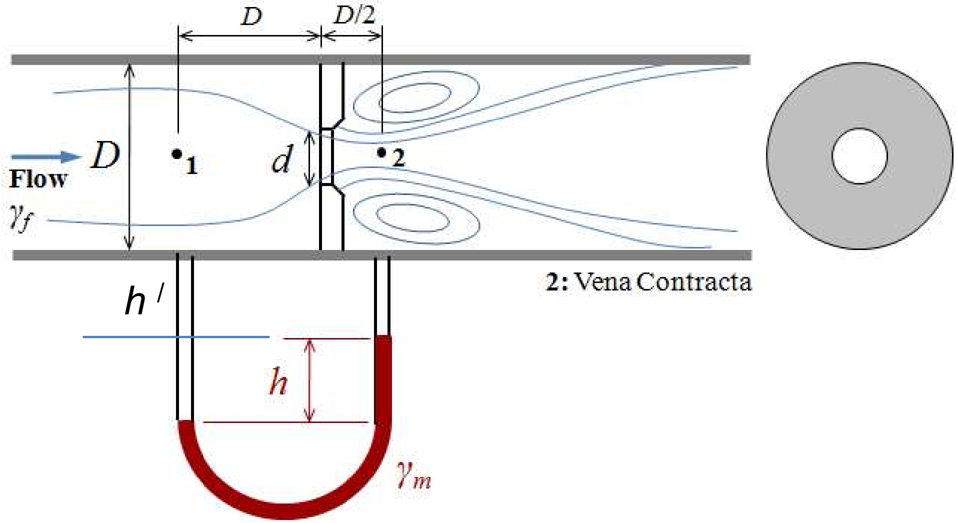
The consideration of vena contracta (point 2) is accommodated by measuring the downstream pressure at point 2 (NOT at the orifice).

Orifice meter

Frictional (viscous) effect becomes very important while such obstruction meter is used in flow systems. To accommodate such effect, empirical **discharge coefficient, C_d** is defined as:

$$C_d = \frac{Q_{actual}}{Q_{theo.}}$$

$$\therefore Q_{actual} = C_d Q_{theo.} = \frac{C_d \; A_0}{\sqrt{\left(1-\beta^4\right)}} \sqrt{\frac{2\left(p_1 - p_2\right)}{\rho_f}} \qquad (v)$$



Pressure differential can be measured using different approaches; for example using U-tube differential manometer as:

$$p_1 + \left(h' + h\right)\gamma_f = p_2 + h'\gamma_f + h\gamma_m$$

$$\Rightarrow p_1 + h\gamma_f = p_2 + h\gamma_m$$

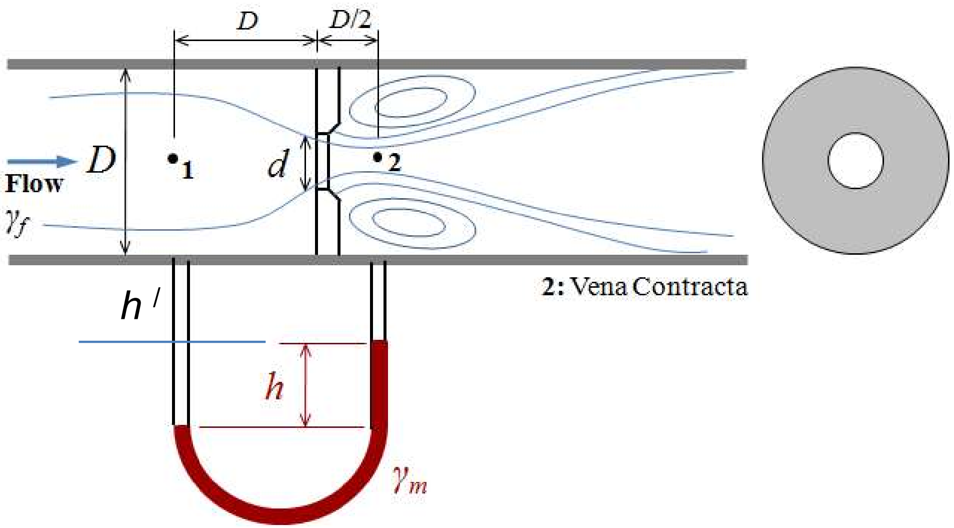
$$\Rightarrow p_1 - p_2 = h\left(\gamma_m - \gamma_f\right)$$

$$\Rightarrow \frac{p_1 - p_2}{\rho_f} = h\left(\frac{\gamma_m - \gamma_f}{\rho_f}\right) \qquad \Rightarrow \frac{p_1 - p_2}{\rho_f} = gh\left(\frac{\rho_m}{\rho_f} - 1\right)$$

specific gravity (SG), S

$$\Rightarrow \frac{p_1 - p_2}{\rho_f} = gh\left(\frac{S_m}{S_f} - 1\right)$$

$$\therefore Q_{actual} = C_d Q_{theo.} = \frac{C_d\ A_0}{\sqrt{(1-\beta^4)}}\sqrt{\frac{2(p_1-p_2)}{\rho_f}} \qquad (v)$$



$$\therefore Q_{actual} = \frac{C_d\ A_0}{\sqrt{(1-\beta^4)}}\sqrt{2gh\left(\frac{S_m}{S_f}-1\right)}$$

$$\therefore Q_{actual} = \frac{C_d\ A_0}{\sqrt{(1-\beta^4)}}\sqrt{2gH}$$

where $H = h\left(\dfrac{S_m}{S_f}-1\right)$

S_f is the specific gravity (SG) of the flowing fluid

S_m is the specific gravity (SG) of the manometric fluid

In short form:

$$Q_{actual} = K\sqrt{2gH} \quad ; \quad K = \frac{C_d\ A_0}{\sqrt{(1-\beta^4)}}$$

$$\text{or,}\quad Q_{actual} = K_1\sqrt{H} \quad ; \quad K_1 = \frac{C_d\ A_0}{\sqrt{(1-\beta^4)}}\sqrt{2g}$$

Orifice meter

ASME recommends the use of curve-fit formula for C_d developed by ISO according to:

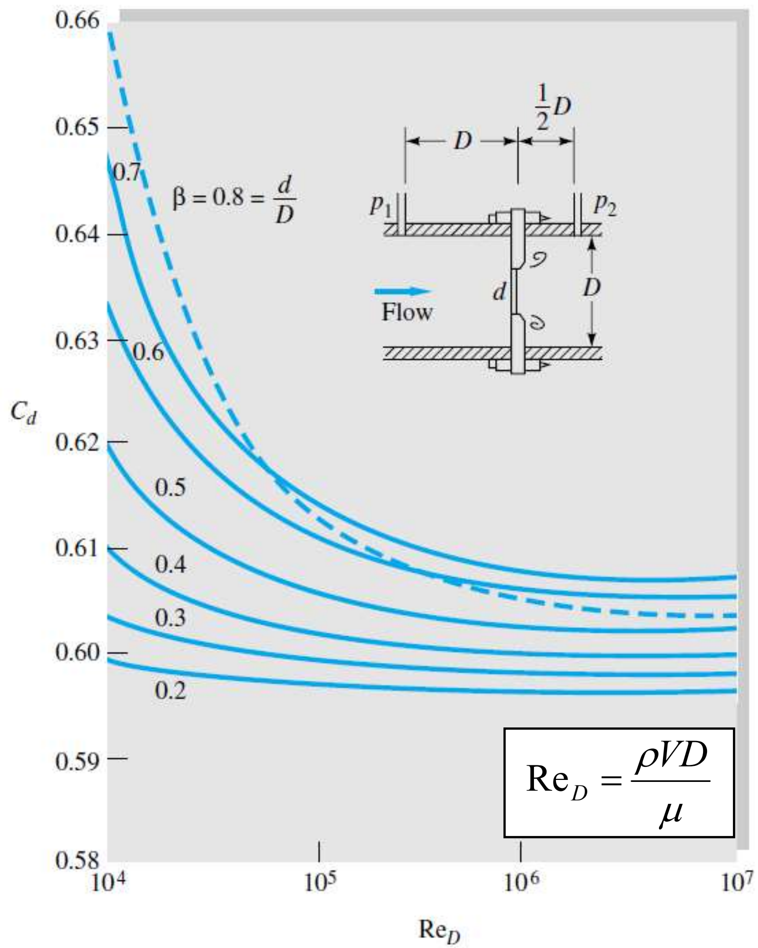
$$C_d = f(\mathrm{Re}_D, \beta)$$

$$C_d = f(\beta) + 91.71\beta^{2.5}\,\mathrm{Re}_D^{-0.75} + \frac{0.09\beta^4}{1-\beta^4}F_1 - 0.0337\beta^3 F_2$$

where:

$$f(\beta) = 0.5959 + 0.0312\beta^{2.1} - 0.184\beta^8$$

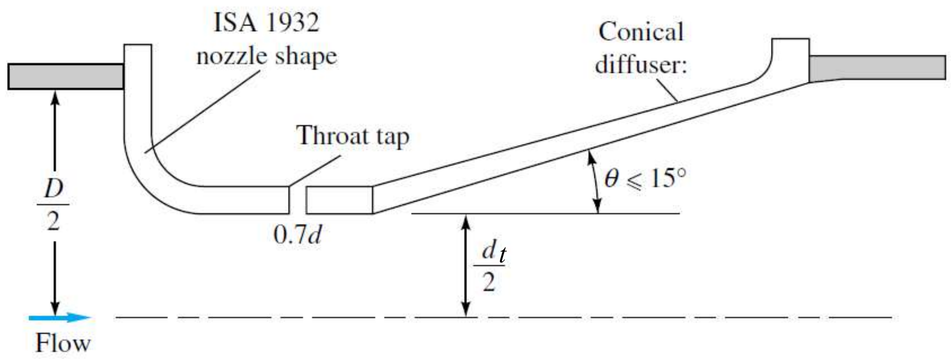
$$F_1 = 0.4333 \qquad F_2 = 0.47 \qquad \left(D : \frac{1}{2}D\,\text{taps}\right)$$



Discharge coefficient of an orifice in the range of Reynolds number 10^4 to 10^7

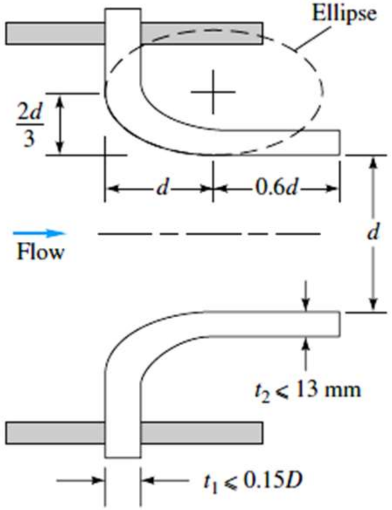
Venturi meter

Modern venuri meter consists of an ISA 1932 nozzle entrance and a conical expansion of half angle no greater than 15 deg. Its discharge coefficient is given by the ISO correlation formula:

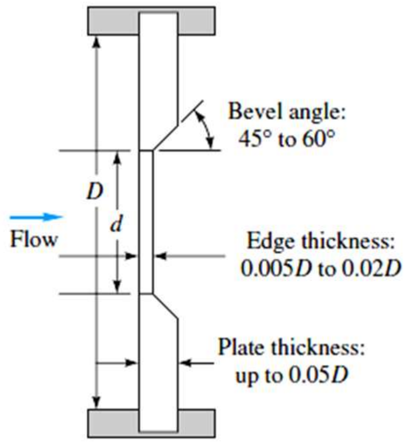


$$C_d \approx 0.9858 - 0.196\beta^{4.5}$$

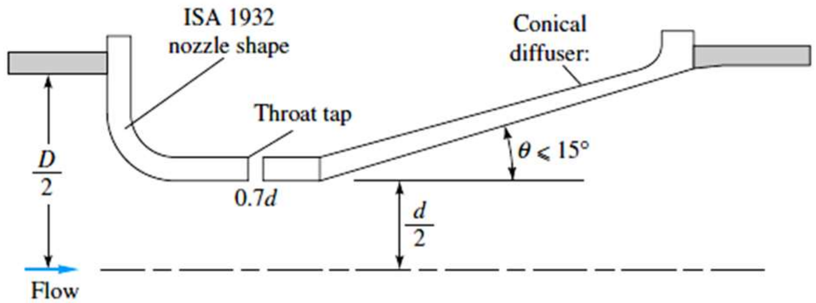
$$\beta = \frac{d_t}{D}$$



(a) **Flow nozzle**



(b) **Orifice**



(c) **Venturi meter**

Type of meter	Net head loss	Cost
Orifice	Large	Small
Nozzle	Medium	Medium
Venturi	Small	Large

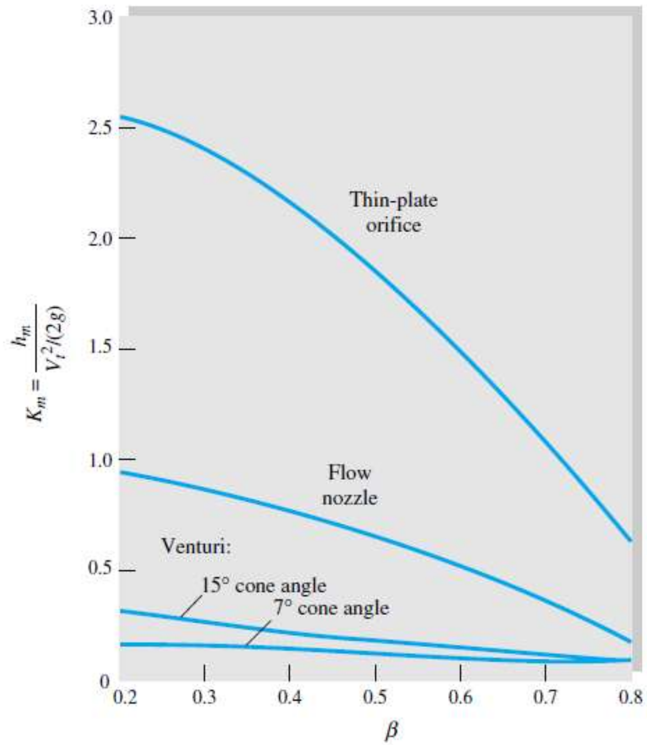


Fig. 6.43 Nonrecoverable head loss in Bernoulli obstruction meters. *(Adapted from Ref. 30.)*

Pitot tube

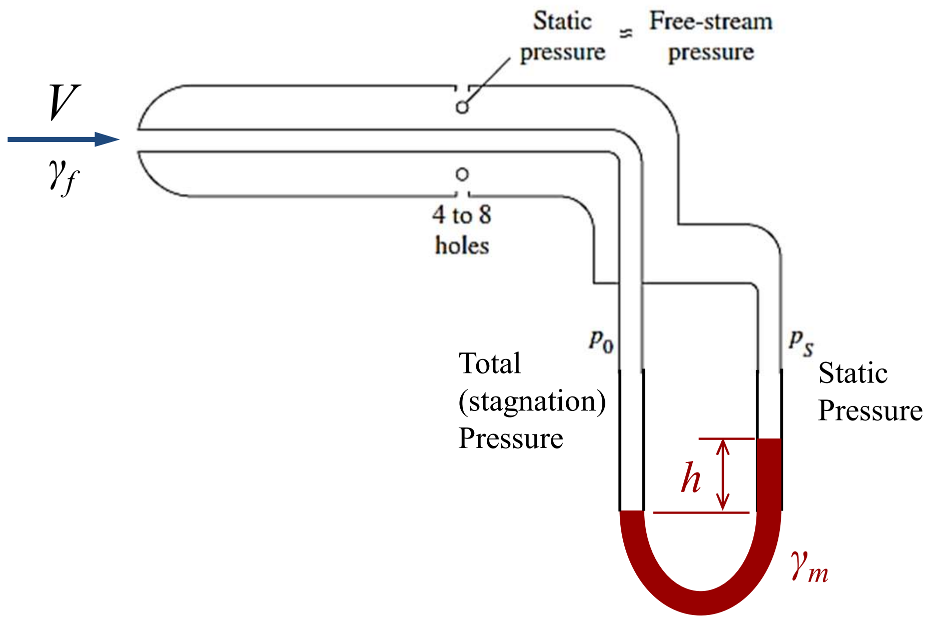
For local velocity measurement inside duct, wind tunnel etc.

Apply Bernoulli equation between stagnation point and static point:

$$\frac{p_0}{\gamma_f}+\frac{V_0^2}{2g}+z_0=\frac{p_s}{\gamma_f}+\frac{V^2}{2g}+z_s$$

$$\Rightarrow \frac{p_0}{\gamma_f}=\frac{p_s}{\gamma_f}+\frac{V^2}{2g}$$

$$\Rightarrow V=\sqrt{\frac{2(p_0-p_s)}{\rho_f}}$$



Pitot tube (pitot-static tube)

Pitot tube

Now from principle of manometry

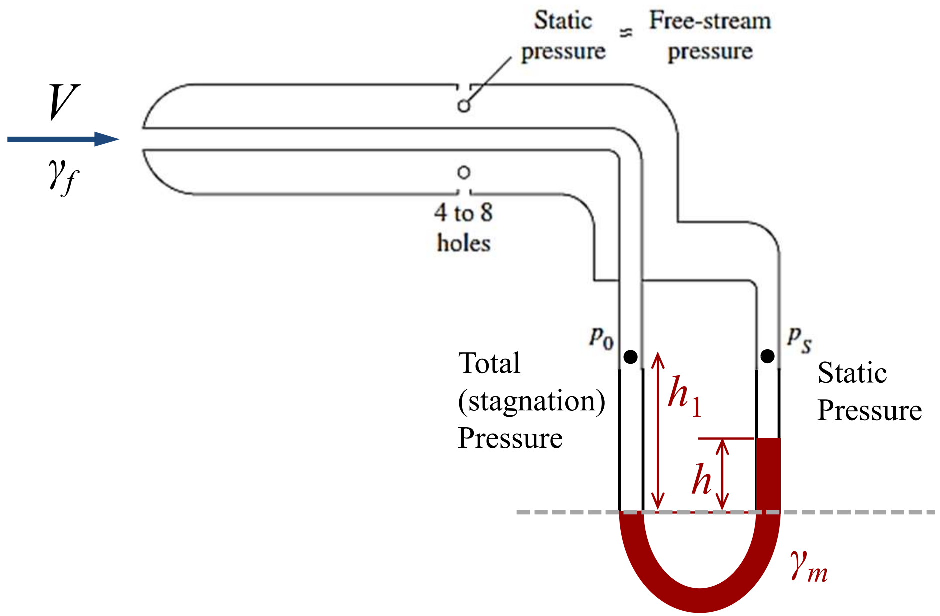
$$p_0 + \gamma_f h_1 = p_s + \gamma_f (h_1 - h) + \gamma_m h$$

$$\Rightarrow p_0 = p_s - \gamma_f h + \gamma_m h$$

$$\Rightarrow p_0 - p_s = h(\gamma_m - \gamma_f)$$

$$\Rightarrow p_0 - p_s = \gamma_f h \left(\frac{\gamma_m}{\gamma_f} - 1 \right)$$

$$\Rightarrow p_0 - p_s = \gamma_f h \left(\frac{S_m}{S_f} - 1 \right)$$



Pitot tube (pitot-static tube)

So, the velocity can be measured as follows:

$$\Rightarrow V = \sqrt{\frac{2(p_0 - p_s)}{\rho_f}}$$

$$\Rightarrow V = \sqrt{2g \left(\frac{S_m}{S_f} - 1 \right) h}$$

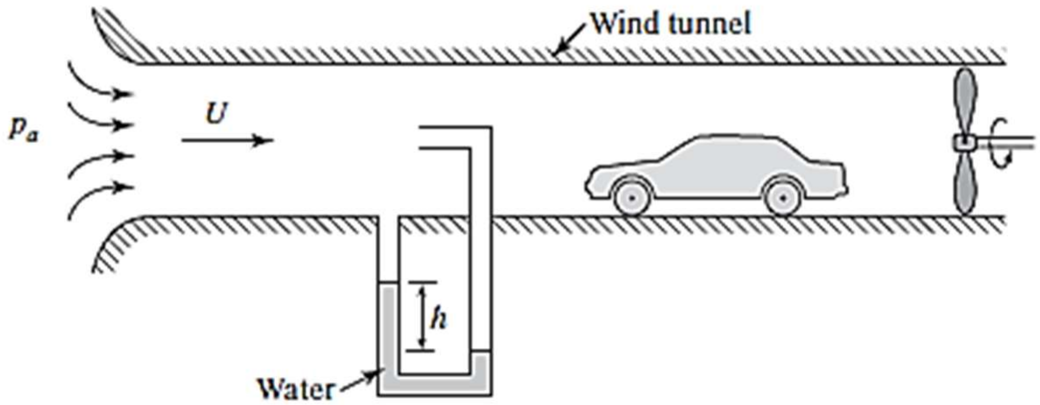
m: manometric fluid
f: flowing fluid

Problem

A tube is used to measure the total pressure inside a wind tunnel as shown in figure. If the water column height in the manometer h = 8 cm, calculate the velocity at the test section.

If the surface area of the car is 5 m^2 and measured drag force is 800 N, determine the drag coefficient at this condition.

Hint: $C_D = \dfrac{F_D}{\frac{1}{2}\rho V^2 A}$



Open Channel Flow measurement

Rectangular notch of size L x H:

Velocity at the infinitesimal element is: $v = \sqrt{2gh}$

Flowrate through the infinitesimal element is:

$$dQ_{theo.} = (Ldh)\sqrt{2gh}$$

Then the actual discharge through the element becomes:

$$dQ_{actual} = C_d dQ_{theo.} = C_d (Ldh)\sqrt{2gh}$$

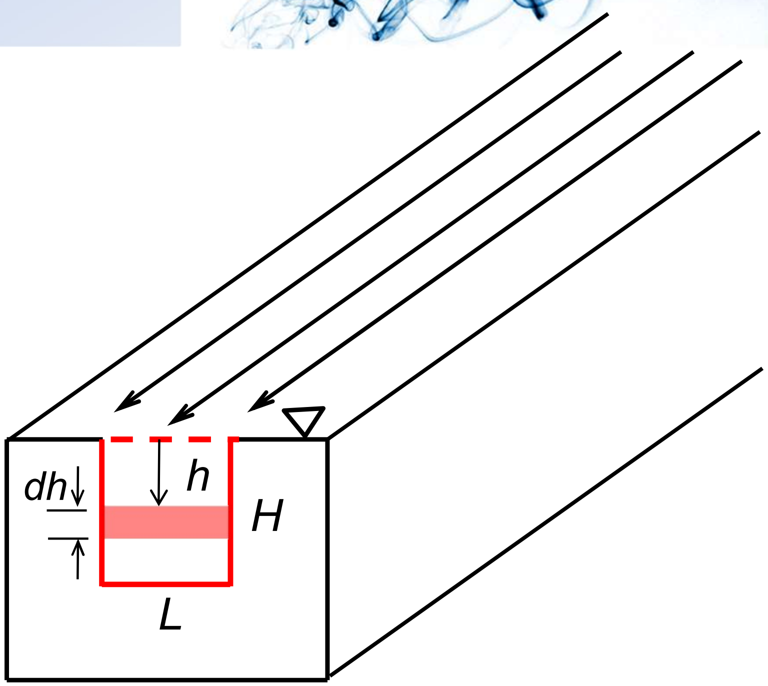
Then the total actual discharge :

$$Q_{actual} = \int_0^H dQ_{actual}$$

$$\Rightarrow Q_{actual} = \int_0^H C_d (Ldh)\sqrt{2gh}$$

$$\Rightarrow Q_{actual} = C_d L \int_0^H \sqrt{2gh}\, dh$$

$$\therefore Q_{actual} = \frac{2}{3}\sqrt{2g} C_d L H^{\frac{3}{2}}$$



Open Channel Flow measurement

Triangular notch :

Width of the strip: $2(H-h)\tan\left(\frac{\theta}{2}\right)$

Velocity at the infinitesimal element is: $v=\sqrt{2gh}$

Flowrate through the infinitesimal element is:

$$dQ_{theo.}=\left(2(H-h)\tan\left(\frac{\theta}{2}\right)dh\right)\sqrt{2gh}$$

Then the actual discharge through the element becomes:

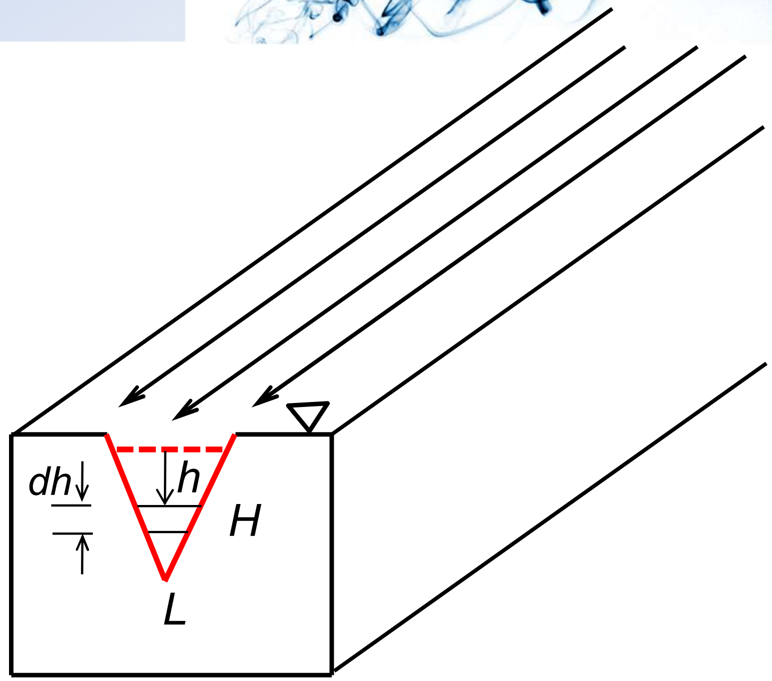
$$dQ_{actual}=C_d\,dQ_{theo.}=C_d\,2(H-h)\tan\left(\frac{\theta}{2}\right)dh\sqrt{2gh}$$

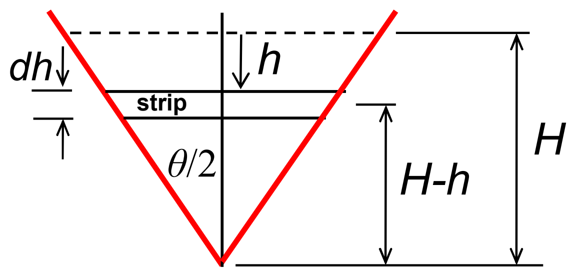
Then the total actual discharge :

$$Q_{actual}=\int_0^H dQ_{actual}$$

$$\Rightarrow Q_{actual}=C_d\int_0^H 2(H-h)\tan\left(\frac{\theta}{2}\right)dh\sqrt{2gh}$$

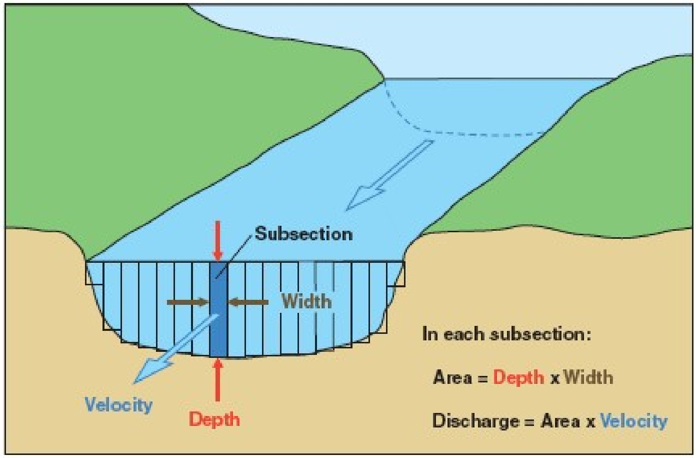
$$\therefore Q_{actual}=\frac{8}{15}\sqrt{2g}\,C_d\tan\left(\frac{\theta}{2}\right)H^{\frac{5}{2}}$$





Symmetric triangular notch

Current Flow meter



Current-meter discharge measurements are made by determining the discharge in each subsection of a channel cross section and summing the subsection discharges to obtain a total discharge.